

- 2 -

$$= \frac{1}{2} \sum_{s=0}^N s x^s = \frac{1-x}{1-x^{N+1}} (0 + x + 2x^2 + 3x^3 + \dots)$$

Now, we can evaluate this sum by a well-known trick: Consider

$$\frac{1}{1-x} = f(x) = \sum_{s=0}^{\infty} x^s, \text{ differentiate } f'(x) = \sum_{s=0}^{\infty} s x^{s-1} = \frac{1}{x} \sum_{s=0}^{\infty} s x^s, \text{ etc.}$$

But we are asked for an approximation as $x \rightarrow 0$, so instead we expand everything in Taylor series:

$$\frac{1}{1-x^{N+1}} = (1-x^{N+1})^{-1} = \left\{ \text{using formula } (1+\varepsilon)^p = 1+p\varepsilon + O(\varepsilon^2) \right\}$$

$$= 1 + x^{N+1} + O((x^{N+1})^2) = 1 + O(x^{N+1})$$

$$0 + x + 2x^2 + \dots = x + O(x^2), \quad 1-x = 1 + O(x)$$

$$\text{Hence, we get: } \langle A \rangle = (1 + O(x))(1 + O(x^{N+1}))(x + O(x^2)) =$$

$$= (1 + O(x) + O(x^{N+1}) + O(x^{N+2}))(x + O(x^2)) =$$

$$= (1 + O(x))(x + O(x^2)) = x + xO(x) + O(x^2) + O(x^3) =$$

$$= x + O(x^2) \approx x = e^{-\varepsilon/\tau}$$

$$\langle A \rangle \approx e^{-\varepsilon/\tau}$$

average
of open links for $\varepsilon \gg \tau$

Three remarks: 1). If you cannot follow the last \mathbb{P} calculation, you need to review Taylor series and related topics. You can come see me and I'll be happy to explain what's going on.

2). It would be a mistake setting $\varepsilon/\tau \rightarrow \infty$. When they ask for a limit $\varepsilon \gg \tau$ they assume ε/τ is large ($\gg 1$) but not infinite. In particular, this means you can ignore terms in ~~a~~ a sum, which are more than ε/τ times smaller than other terms. In other words, normally when